NAG Toolbox for MATLAB

f08jb

1 Purpose

f08jb computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

2 Syntax

```
[d, e, m, w, z, jfail, info] = f08jb(jobz, range, d, e, vl, vu, il, iu,
abstol, 'n', n)
```

3 Description

f08jb computes the required eigenvalues and eigenvectors of A by reducing the tridiagonal matrix to diagonal form using the QR algorithm. Bisection is used to determine selected eigenvalues.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Demmel J W and Kahan W 1990 Accurate singular values of bidiagonal matrices *SIAM J. Sci. Statist. Comput.* **11** 873–912

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz – string**

If **jobz** = 'N', compute eigenvalues only.

If **jobz** = 'V', compute eigenvalues and eigenvectors.

Constraint: **jobz** = 'N' or 'V'.

2: **range – string**

If **range** = 'A', all eigenvalues will be found.

If **range** = 'V', all eigenvalues in the half-open interval $(\mathbf{vl}, \mathbf{vu}]$ will be found.

If **range** = 'I', the **il**th to **iu**th eigenvalues will be found.

Constraint: **range** = 'A', 'V' or 'I'.

3: $\mathbf{d}(*)$ **– double array**

Note: the dimension of the array **d** must be at least $\max(1, \mathbf{n})$.

The n diagonal elements of the tridiagonal matrix A.

4: **e($*$) – double array**

Note: the dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$.

The $(n - 1)$ subdiagonal elements of the tridiagonal matrix A.

5: **vl – double scalar**

6: **vu – double scalar**

If **range** = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.

If **range** = 'A' or 'I', **vl** and **vu** are not referenced.

Constraint: if **range** = 'V', **vl** < **vu**.

7: **il – int32 scalar**

8: **iu – int32 scalar**

If **range** = 'I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned.

If **range** = 'A' or 'V', **il** and **iu** are not referenced.

Constraints:

if $\mathbf{n} = 0$, $\mathbf{il} = 1$ and $\mathbf{iu} = 0$;
if $\mathbf{n} > 0$, $1 \leq \mathbf{il} \leq \mathbf{iu} \leq \mathbf{n}$.

9: **abstol – double scalar**

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a, b]$ of width less than or equal to

$$\mathbf{abstol} + \epsilon \max(|a|, |b|),$$

where ϵ is the ***machine precision***. If **abstol** is less than or equal to zero, then $\epsilon \|A\|_1$ will be used in its place. Eigenvalues will be computed most accurately when **abstol** is set to twice the underflow threshold $2 \times$ x02am( ), not zero. If this function returns with **info** > 0, indicating that some eigenvectors did not converge, try setting **abstol** to $2 \times$ x02am( ). See Demmel and Kahan 1990.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **d**.

n, the order of the matrix.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldz, work, iwork

5.4 Output Parameters

1: **d($*$) – double array**

Note: the dimension of the array **d** must be at least $\max(1, \mathbf{n})$.

May be multiplied by a constant factor chosen to avoid over/underflow in computing the eigenvalues.

2: **e($*$) – double array**

Note: the dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$.

May be multiplied by a constant factor chosen to avoid over/underflow in computing the eigenvalues.

3: **m – int32 scalar**

The total number of eigenvalues found.

If **range** $=$ 'A', $\mathbf{m} = \mathbf{n}$.

If **range** $=$ 'V', the exact value of **m** is not known in advance, but will satisfy $0 \leq \mathbf{m} \leq \mathbf{n}$.

If **range** $=$ 'I', $\mathbf{m} = \mathbf{iu} - \mathbf{il} + 1$.

4: **w**$(*)$ **– double array**

Note: the dimension of the array **w** must be at least $\max(1, \mathbf{n})$.

The first **m** elements contain the selected eigenvalues in ascending order.

5: **z**(**ldz**,$*$) **– double array**

The first dimension, **ldz**, of the array **z** must satisfy

if **jobz** $=$ 'V', **ldz** $\geq \max(1, \mathbf{n})$;
ldz ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{m})$

If **jobz** $=$ 'V', then if **info** $= 0$, the first m columns of Z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the ith column of Z holding the eigenvector associated with $\mathbf{w}(i)$.

If an eigenvector fails to converge (**info** > 0), then that column of Z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in **jfail**.

If **jobz** $=$ 'N', **z** is not referenced.

Note: you must ensure that at least $\max(1, \mathbf{m})$ columns are supplied in the array **z**; if **range** $=$ 'V', the exact value of **m** is not known in advance and an upper bound must be used.

6: **jfail**$(*)$ **– int32 array**

Note: the dimension of the array **jfail** must be at least $\max(1, \mathbf{n})$.

If **jobz** $=$ 'V', then if **info** $= 0$, the first **m** elements of **jfail** are zero.

If **info** > 0, **jfail** contains the indices of the eigenvectors that failed to converge.

If **jobz** $=$ 'E', **jfail** is not referenced.

7: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **range**, 3: **n**, 4: **d**, 5: **e**, 6: **vl**, 7: **vu**, 8: **il**, 9: **iu**, 10: **abstol**, 11: **m**, 12: **w**, 13: **z**, 14: **ldz**, 15: **work**, 16: **iwork**, 17: **jfail**, 18: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, then i eigenvectors failed to converge. Their indices are stored in array **jfail**. Please see **abstol**.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^2 if **jobz** = 'N' and is proportional to n^3 if **jobz** = 'V' and **range** = 'A', otherwise the number of floating-point operations will depend upon the number of computed eigenvectors.

9 Example

```
jobz = 'Vectors';
range = 'Values in range';
d = [1;
     4;
     9;
     16];
e = [1;
     2;
     3];
vl = 0;
vu = 5;
il = int32(0);
iu = int32(0);
abstol = 2*x02am();
[dOut, eOut, m, w, z, jfail, info] = f08jb(jobz, range, d, e, vl, vu, il,
iu, abstol)
```

```
dOut =
     1
     4
     9
    16
eOut =
     1
     2
     3
m =
           2
w =
    0.6476
    3.5470
         0
         0
z =
    0.9396    0.3388
   -0.3311    0.8628
    0.0853   -0.3648
   -0.0167    0.0879
jfail =
           0
           0
           0
           0
```

```
info =
          0
```